Exponenciální rovnice - rce s neznámou v exponentu

$$2^x + 4^x = 16$$

Způsoby řešení exponenciálních rovnic:

a) $a^x = a^y \Rightarrow x = y$

b) substitucí

c) vytýkáním

d) logaritmicky

Řešené příklady:

$$2^{2x} = 16$$

$$2^{2x} = 2^4$$

$$2x = 4$$

$$x = 2$$

NSolve[2^{2x} == 16, x]

{x → 2.}

$$5^{x-4} = 0,\ 008$$

$$5^{x-4} = \frac{8}{1000}$$

$$5^{x-4} = \frac{1}{125}$$

$$5^{x-4} = 5^{-3}$$

$$x - 4 = -3$$

$$x = 1$$

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NSolve[5^(x-4) == 0.008, x]
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{{x → 1.}}
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$$\left(\frac{4}{25}\right)^{x+3} \cdot \left(\frac{125}{8}\right)^{4x-1} = \frac{5}{2}$$

$$\left(\frac{5}{2}\right)^{-2x-6} \cdot \left(\frac{5}{2}\right)^{12x-3} = \frac{5}{2}$$

$$\left(\frac{5}{2}\right)^{10x-9} = \left(\frac{5}{2}\right)$$

$$10x - 9 = 1$$

$$10x = 10$$

$$x = 1$$

$$\text{NSolve}\left[\left(\frac{4}{25}\right)^{x+3} * \left(\frac{125}{8}\right)^{4x-1} == \frac{5}{2}, x\right]$$

$$\{\{x \to 1.\}\}$$

$$\sqrt[2x+7]{4^{13-x}} = 1024$$

$$4^{\frac{(13-x)}{2x+7}} = 2^{10}$$

$$2^{2\frac{(13-x)}{2x+7}} = 2^{10}$$

$$\frac{(26-2x)}{2x+7} = 10$$

$$26 - 2x = 20x + 70$$
$$-44 = 22x$$
$$x = -2$$

FindRoot[$\sqrt[2*x+7]{4^{13-x}}$ == 1024, {x, 0}]

{x → -2.}

Úlohy k řešení:

(kontrolu správnosti proveďte v programu Mathematica)

a) $\left(1-\frac{5}{9}\right)^{\frac{2}{3-2x}}=\left(\frac{9}{4}\right)^{\frac{3}{x-3}}$

b) $\frac{10^{x^2}}{2^{-15}}=\frac{5^{-15}}{10^{12-12x}}$

c) $\left(\frac{4}{9}\right)^{x}\cdot\left(\frac{27}{8}\right)^{x-1}=\frac{2}{3}$

d) $27^{5x-6}.81^{2x+3}=9^{4x-2}.3^{7x-2}$

Výsledky:

In[86]:= NSolve$\left[\left(1-\frac{5}{9}\right)^{\frac{2}{3-2x}} == \left(\frac{9}{4}\right)^{\frac{3}{x-3}}, x\right]$ Out[86]= $\{\{x \to 0.75\}\}$

In[85]:= NSolve$\left[\frac{10^{x^2}}{2^{-15}} == \frac{5^{-15}}{10^{12-12x}}, x\right]$ Out[85]= $\{\{x \to 3.\}, \{x \to 9.\}\}$

In[88]:= NSolve$\left[\left(\frac{4}{9}\right)^{x} * \left(\frac{27}{8}\right)^{x-1} == \frac{2}{3}, x\right]$ Out[88]= $\{\{x \to 2.\}\}$

In[89]:= NSolve$\left[27^{5x-6} * 81^{2x+3} == 9^{4x-2} * 3^{7x-2}, x\right]$ $\{x \to 0.\}$

Exponenciální rovnice 2:

$$4^{x+1} - 8 * 4^{x-1} = 32$$

$$4^{x-1}\left(4^{2} - 8\right) = 32$$

$$4^{x-1} .8 = 32 \ /8$$

$$4^{x-1} = 4$$

$$x - 1 = 1$$

$$x = 2$$

NSolve[$4^{x+1} - 8 * 4^{x-1}$ == 32, x] {{x → 2.}}

JINÝ ZPŮSOB ŘEŠENÍ:

$$a^{x+y} = a^x + a^y$$

$$4^{x+1} - 8 * 4^{x-1} = 32$$

$$a^{x-y} = \frac{a^x}{a^y}$$

$$4 * 4^x - 8 * \frac{4^x}{4} = 32$$

$$4 * 4^x - 2 * 4^x = 32$$

$$2 * 4^x = 32 \;/ : 2$$

$$4^x = 16$$

$$x = 2$$

$$3^{2x-1} + 3^{2x-2} - 3^{2x-4} = 315$$

$$3^{2x-4}(3^3 + 3^2 - 1) = 315$$

$$3^{2x-4} * 35 = 315 \ /: 35$$

$$3^{2x-4} = 9$$

$$2x - 4 = 2$$

$$x = 3$$

NSolve[$3^{2x-1} + 3^{2x-2} - 3^{2x-4} == 315, x$]

{{x → 3.}}

Příklady k procvičení:

(zkoušku proveďte výpočtem v programu Mathematica)

a) $5 * 4^{x+1} - 4^{x+2} = 4^{x-1} + 240$

b) $5 * 2^{x+2} - 6 * 3^{x+2} = 3^{x+3} + 2 * 2^{x+1}$

c) $8^{x-1} - 3 = 7 * 8^{x-2} + 5$

d) $5^{x} + 1 = 3 * 5^{x-1} + 11$

Výsledky úloh

a)

In[5]:= $5*4^{x+1}-4^{x+2}=4^{x-1}+240$

NSolve[$5*4^{x+1}-4^{x+2}==4^{x-1}+240$, x]

Out[6]= {{x → 3.}}

b)

In[7]:= NSolve[$5*2^{x+2}-6*3^{x+2}==3^{x+3}+2*2^{x+1}$, x]

Out[7]= {{x → -4.}}

c)

In[9]:= NSolve[$8^{x-1}-3==7*8^{x-2}+5$, x]

Out[9]= {{x → 3.}}

d)

NSolve[$5^{x}+1==3*5^{x-1}+11$, x]

{{x → 2.}}

Exponenciální rovnice řešené substitucí

$$3^{2x-1} + 3^{x} - 3^{0} = 3^{-1}$$

$$\frac{3^{2x}}{3} + 3^{x} - 1 = \frac{1}{3} \; / *3$$

$$3^{2x} + 3 * 3^{x} - 3 = 1$$

$$3^{x} = t$$

$$t^{2} + 3t - 4 = 0$$

$$t_1 = -4 \qquad t_2 = 1$$

$$3^{x} = -4 \quad \Rightarrow P = \emptyset$$

$$3^{x} = 1 \;\Rightarrow\; x = 0$$

$$\text{NSolve}[3^{2x-1} + 3^{x} - 3^{0} == 3^{-1}, x]$$

$$\{x \to 0.\}$$

$$25^{2x} - 3 * 25^{x} = 10$$

$$25^{2x} - 3 * 25^{x} = 10$$

$$25^{x} = t$$

$$t^{2} - 3t - 10 = 0$$

$$t_1 = 5 \quad t_2 = -2$$

$$25^{x} = 5 \Rightarrow 5^{2x} = 5 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$$

$$25^{x} = -2 \Rightarrow P = \emptyset$$

NSolve[$25^{2x} - 3 * 25^{x} == 10, x$]

{x → 0.5

Řešte v R, zkoušku proveďte v programu Math.

a) $4^x - 10 * 2^{x-1} = 24$

b) $\sqrt[x]{81} + \dfrac{27}{\sqrt[x]{81}} = 12$

c) $9^{\sqrt{x+2}} = 27 * 3^{\sqrt{x+2}}$

Výsledky:

a) $\text{NSolve}[2^{2x} - 10 * 2^{x-1} == 24, x]$ $\{x \to 3.\}$

b) $\text{NSolve}\left[\sqrt[x]{81} + \frac{27}{\sqrt[x]{81}} == 12, x\right]$ $\{x \to 2.\}, \{x \to 4.\}$

c) $\text{NSolve}\left[9^{\sqrt{x+2}} == 27 * 3^{\sqrt{x+2}}, x\right]$ $\{\{x \to 7.\}\}$

Různé úlohy:

$$2^{3x-1} * 4 = 8^{x+1} * 0.5^{x}$$

{{x → 2.}}

$$\frac{81}{16} = \left(\frac{2}{3}\right)^{x} * \left(\frac{9}{4}\right)^{x+1}$$

{{x → 2.}}

$$3^{x} + 3^{x+1} = 108$$

{{x → 3.}}

$$6 * 7^{x+3} - 7^{x+2} = 41$$

{{x → -2.}}

$$9 * 3^{x} + 3^{-x} = 10$$

$\{\{x \to -2.\}, \{x \to 0.\}\}$

$$16^{x} = 8 * 4^{x} + 2 * 8^{x}$$

$\{x \to 2.\}$

$$2^{x-1} - 2^{x-2} = 5^{x-3} + 2^{x-3}$$

$\{\{x \to 3.\}\}$

$$3^{x+1} + 2 * 3^{-x} = 7$$

$\{\{x \to -1.\}, \{x \to 0.63093\}\}$

$$2^{x} * 3^{x-1} = 12$$

$\{\{x \to 2.\}\}$